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Instantaneous Speed Ratio of Traffic Flowing through a Merging Area at Kilometer 31.6 on the Highway from Shah Alam to Kuala Lumpur

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ABSTRACT

This study aims to evaluate a continuous flow model that involves a ramp area at kilometer 31.6 on the highway from Shah Alam to Kuala Lumpur, to analyze the findings of numerical results of instantaneous speed ratios and to observe the convergence patterns for each section. The continuous flow model assumes traffic flow to be similar to the heat equation in regard to the concept of the one-dimensional viscous flow of a compressible fluid. For the methodology, for solving an initial value-boundary value problem, an initial condition together with a set of boundary conditions are required to solve the partial differential equation. The boundary conditions are chosen to assess the suitableness of the design of the entrance ramp in Malaysia, which is for right hand drive traffic. Highway traffic data were collected on the tapered acceleration lane and obtained by the videotaping method. The Maple programming language was used to write a numerical code in order to evaluate the instantaneous speed ratio in terms of a Fourier series. Our results show that the realistic results of instantaneous speed ratios on the ramp at kilometer 31.6 from Shah Alam to Kuala Lumpur are acceptable when compared to the theoretical results. Therefore, a very minimal collision rate is expected due to the well-designed

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ramp at kilometer 31.6 from Shah Alam to Kuala Lumpur. It is beneficial to study the mathematical model and theories of traffic flows on the merging area to enhance the efficiency of the traffic flowing on highways.

Keywords: Continuous flow model, heat equation, highway operation, macroscopic model, partial differential equation

INTRODUCTION

Presently, highways are the main road system in the movement of traffic and goods. Highways have provided the motorist with a high level of service. In the 1950s, the highway concept appeared from the roadway concept (Adnan, 2007). Many researchers and highway planners in the field of highway design and traffic engineering have shown their interest in cars following theories and models.

Macroscopic models of traffic flow consider the flow of traffic to be similar to the physical flow of fluid (Lazar et al., 2016). Traffic dynamics are described as a function of space and time corresponding to the partial differential equation of aggregate macroscopic quantities such as traffic density, traffic average flow or velocity (Lazar et al., 2017). As mentioned by Drew (1964), continuous flow models are realistic when applied to freeway traffic flow.

The available research on highways mostly focuses on physical studies rather than evaluating mathematical models. In addition, the research develops car theories and models macroscopically for traffic that is left hand drive, for example Lighthill and Witham (1955), Harr and Leornads (1962) and Drew (1964). Research by Reddy (1966) based the theory of traffic flow on the one-dimensional movement of fluid or gas. In the past 20 years, most of the research undertaken focused on developing a new car theory and models, for instance models by Daganzo (2002), Wong and Wong (2002), Mathew (2014) and Van Wageningen-Kessels et al. (2015).

This paper focuses on a continuous flow model for traffic flowing onto the merging area of a ramp at kilometer 31.6 on the highway from Shah Alam to Kuala Lumpur. One of the macroscopic models in use is the continuous flow model which originated from the hydrodynamic analogy of vehicle flow. This is based on the assumption that the traffic flow on highways operates in the same way as a one-dimensional viscous flow of compressible fluid. The parameters that are used in this paper are instantaneous speed ratio and easiness to flow F_0. Both parameters are important in evaluating the continuous flow model.

Reddy (1966) took the Fourier series solution to the first iterations only. We tried to set the equation restricted to a number of iterations, n. We used the number of iterations n instead of tolerance as a stopping criteria because we wanted to see the pattern of the convergence for every selected easiness to flow value, F_0. Reddy (1966) used the parameter easiness to flow F_0 as a dependent variable, which is similar to our study. This means it is appropriate to compare our results with the theoretical results stated by Reddy (1966).

The motivating pressure potential P corresponds to the fluid dynamic relationship between flow velocity and potential velocity. Hence, the velocity of vehicles between two points depends on the difference in potential at these points (Harr & Leonards, 1962). Thus, we assume that the motivating pressure potential P is similar to the theory of one-dimensional viscous flow of compressible fluid.

The findings of this study will give a better understanding of the continuous flow model as well as methods for improving the design of the entrance ramp at kilometer 31.6 on the highway from Shah Alam to Kuala Lumpur. The results obtained are important for evaluation and decision-making relating to traffic flow ramp design.

The objectives of this paper are (a) to evaluate the continuous flow model by determining the instantaneous speed ratio for every selected value of the parameter easiness to flow F_0 on a ramp area at kilometer 31.6 from Shah Alam to Kuala Lumpur,(b) to analyze the findings of the numerical results of the instantaneous speed ratios for every selected easiness to flow F_0 value, and (c) to observe the pattern of the convergence for every selected easiness to flow F_0 value.

METHODS

These notations are used throughout the paper.

P = Motivating pressure potential,
X = A specific position along the highway,
L = A particular length of highway upstream of the merging point,
V = Vehicle speed,
t = Time,
Q = Highway volume per time t,
R_w = Number of ramp vehicles merging per time t,
R_v = Number of ramp vehicles merging per length L,
$\frac{V(X,t)}{\bar{V}}$ = Ration of instantaneous speed at a point X, to the average of length L,
F_0 = Parameter easiness to flow,
n = Number of iterations.

The general heat Equation (1) is given as:

$$\frac{\partial^2 P}{\partial X^2} - a^2 \frac{\partial P}{\partial t} = C \qquad (1)$$

where a^2 and C are constant.

A set of boundary conditions is required to solve the Partial Differential Equation (PDE) in (1). Some of the boundary conditions are explained as in Equation (2), (3) and (4).

The driver is ignorant of the changes in the motivating pressure potential ahead on the highway at time $t = 0$.

$$P(X,0) = P_0 \tag{2}$$

Pressure potential changes at the entrance ramp due to the merging of vehicles, which is at distance $X = L$ from initial potential to P_0 to P_1,

$$P(L,t) = P_1 \tag{3}$$

The pressure potential is P_0 and the vehicles are unaffected by the entrance ramp ahead at $X = 0$.

$$P(0,t) = P_0 \tag{4}$$

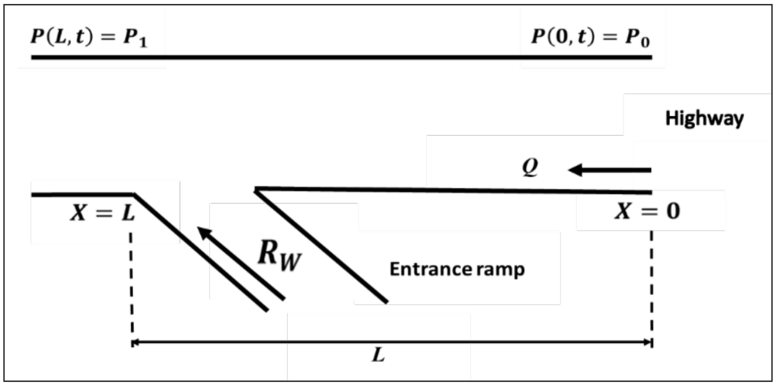

Figure 1. Illustration of the highway and entrance ramp with boundary conditions (BCs).

The mathematical model is represented in the following initial boundary value problem as in Equation (5), (6), (7) and (8).

$$P_{XX} = a^2 P_t + C, \qquad 0 \le X \le L, t > 0 \tag{5}$$

$$P(0,t) = P_0, \qquad t > 0 \tag{6}$$

$$P(L,t) = P_1 \qquad t > 0 \tag{7}$$

$$P(X,0) = P_0 \qquad 0 < X < L \tag{8}$$

The solution of the initial boundary value problem (IBVP), as in Equations (1), (2), (3) and (4), is carried out using the separation of variables (SOV) method as follows:

$$P_{\infty} = \frac{V(X,t)}{\bar{\bar{V}}} = 1 + \frac{R_w}{2QL}(L - 2X) + \sum_{n=1}^{\infty}\left(\frac{0.2R_w L(-1)^n}{Q}\left(-cos\left(\frac{n\pi - 1}{n^2}\right)\right) + 2(-1)^{2n-2}\right)\cdot\ e^{-n^2 F_0}\cdot cos\left(\frac{n\pi X}{L}\right) \tag{9}$$

The Fourier series of the solution as shown in Equation (9) needs to be set with the number of iterations *n* such that the infinite iteration in Equation (9) is restricted to a number of iterations. This is a necessity to handle the time-consuming and memory-consuming process of the numerical computations.

In this step, we want to investigate the number of the iterations needed in order to get the convergence values of instantaneous speed ratio with an acceptable error. We can take the first iteration P_1, the second iteration P_2, third iteration P_3 and so on. The equations after we place the number of iterations *n* in (9) are shown respectively as in Equation (10), (11), (12), (13) and (14):

$$P_1 = \frac{V(X,t)}{\bar{\bar{V}}} = 1 + \frac{R_w}{2QL}(L - 2X) + \left(2 - \frac{0.4R_w}{Q}\right)\left(e^{-F_0}\,cos\left(\frac{\pi X}{L}\right)\right) \tag{10}$$

$$P_2 = \frac{V(X,t)}{\bar{\bar{V}}} = 1 + \frac{R_w}{2QL}(L - 2X) + \left(2 - \frac{0.4R_w}{Q}\right)\left(e^{-F_0}\,cos\left(\frac{\pi X}{L}\right)\right) + \left(2e^{-4F_0}\,cos\left(\frac{2\pi X}{L}\right)\right) \tag{11}$$

$$P_3 = \frac{V(X,t)}{\bar{V}} = 1 + \frac{R_w}{2QL}(L - 2X) + \left(2 - \frac{0.4R_w}{Q}\right)\left(e^{-F_0}cos\left(\frac{\pi X}{L}\right)\right) + \left(2e^{-4F_0}cos\left(\frac{2\pi X}{L}\right)\right) + \left(2 - \frac{0.4R_w}{9Q}\right)\left(e^{-9F_0}cos\left(\frac{3\pi X}{L}\right)\right) \tag{12}$$

$$P_4 = \frac{V(X,t)}{\bar{V}} = 1 + \frac{R_w}{2QL}(L - 2X) + \left(2 - \frac{0.4R_w}{Q}\right)\left(e^{-F_0}cos\left(\frac{\pi X}{L}\right)\right) + \left(2e^{-4F_0}cos\left(\frac{2\pi X}{L}\right)\right) + \left(2 - \frac{0.4R_w}{9Q}\right)\left(e^{-9F_0}cos\left(\frac{3\pi X}{L}\right)\right) + \left(2e^{-16F_0}cos\left(\frac{4\pi X}{L}\right)\right) \tag{13}$$

$$P_5 = \frac{V(X,t)}{\bar{V}} = 1 + \frac{R_w}{2QL}(L-2X) + \left(2 - \frac{0.4R_w}{Q}\right)\left(e^{-F_0} cos\left(\frac{\pi X}{L}\right)\right) + \left(2e^{-4F_0} cos\left(\frac{2\pi X}{L}\right)\right)$$
$$+ \left(2 - \frac{0.4R_w}{9Q}\right)\left(e^{-9F_0} cos\left(\frac{3\pi X}{L}\right)\right) + \left(2e^{-16F_0} cos\left(\frac{4\pi X}{L}\right)\right)$$
$$+ \left(2 - \frac{0.4R_w}{25Q}\right)\left(e^{-25F_0} cos\left(\frac{5\pi X}{L}\right)\right) \tag{14}$$

Highway traffic data were provided by the Faculty of Civil Engineering, Universiti Teknologi Mara (UiTM) under file code 600/IRDC/ST/5/3/1102. The data were collected from a tapered acceleration lane using a videotaping method. This research involves a ramp area at kilometer 31.6 on the highway from Shah Alam to Kuala Lumpur where L (the length of the highway upstream of the merging area point) and X (a specific position along the highway) are varied in 20 m increments.

The Maple programming language (Maple 2017) was used to write a code to evaluate the Fourier series with a finite number of iterations, P_1, P_2, P_3, P_4 and P_5 as Equations (10), (11), (12), (13) and (14). All the values of P_1, P_2, P_3, P_4 and P_5 are instantaneous speed ratios. According to Reddy (1966) the instantaneous speed ratio can be predicted if the accuracy of easiness to flow F_0 is 1 or above. However, the theory of easiness to flow F_0 has never been observed. Reddy (1966) suggested the easiness to flow F_0 computation was restricted to R_w/Q being less than 5.0.

Hence, the values of the easiness to flow parameter F_0 range from 0.1 to 5.0. The location along the highway X/L is plotted on the x-axis and instantaneous speed ratio is plotted on the y-axis.

RESULTS AND DISCUSSION

Instantaneous Speed Ratio and Error $|P_n - P_{n-1}|$

This section discusses the results of the numerical errors $|P_n - P_{n-1}|$ of Equation (9) that describe the instantaneous speed ratio on the ramp at kilometer 31.6 from Shah Alam to Kuala Lumpur. On this specific ramp, we have the values of the volume of traffic on the highway per time Q the ramp vehicles merging per time R_w and the length of section L which are given by 6482 vehicles per hour (vph), 774.656 vph and 170 meters, respectively. Table 1-5 show the numerical results of P_n and the numerical errors $|P_n - P_{n-1}|$ of Equation (9) where $1 \leq n \leq 5$, $n \in$ N for easiness to flow F_0 = 0.50, 0.70, 1.00, 3.00 and 5.00.

The numerical results of P_n are important for us to see the pattern of instantaneous speed ratio along the merging area. These realistic results are necessary to compare with the theoretical results as mentioned in Reddy (1966) (Figure 1). The similarity of the patterns for realistic and theoretical results gives the impression that the design of the ramp on at kilometer 31.6 from Shah Alam to Kuala Lumpur is acceptable and satisfactory. The calculations of errors $|P_n - P_{n-1}|$ are important to analyze how accurate and fast the convergence of solution P_n is for each of the easiness to flow F_0 values. This is essential to observe whether or not the series solution in Equation (9) is a fast converging series.

From our observations, when the easiness to flow F_0 is approaching 5.00, the reading values of instantaneous speed ratio converge faster as shown in Tables 1-5. Note that some of the numerical values of the error for instantaneous speed ratio in Tables 3-5 appear as zero due to the limitation of up to only double precision in computing the numbers.

Instantaneous Speed Ratio–Distance Graphs

In this study, the series solution in Equation (9) is taken up to the fifth iteration P_5. A programming code was written in the Maple programming language (Maple 2017) for the evaluation of the Equations (10), (11), (12), (13) and (14). The results for the instantaneous speed ratio on the ramp at kilometer 31.6 from Shah Alam to Kuala Lumpur will be elaborated upon below.

This section discusses plotted data for the first iteration P_1, second iteration P_2, third iteration P_3, fourth iteration P_4 and fifth iteration P_5 which are presented in Figure 2-4. The instantaneous speed ratio for all of easiness to flow F_0 values continuously decreased from the starting point 0 to the length L. Note that the instantaneous speed ratio is almost static at value 1 whenever the parameter easiness to flow F_0 is approaching 5.00. Note also that the curves are declining with steeper slopes for lower F_0.

As we can see in Figure 2, when the number of iterations n is equal to 1(n = 1), the graph intersects at the middle of L i.e. $X = L/2$. The negative values for the instantaneous speed ratio at specific position X at X=140 meters until X = 170 meters for easiness to flow F_0 = 0.50 and F_0 = 0.70 do not mean the vehicles were traveling in the opposite direction or returning back, since the first iteration P_1 still does not reach the desired solution. We note that as the number of iterations increases (Figure 3-6), all the negative values of the instantaneous speed ratio trend toward values that are ≥ 0. Figure 6 shows the graph of the instantaneous speed ratio versus position X when we increase the number of iterations n to be equal to five n = 5. The values of instantaneous speed ratio do not differ much after n = 5 since the errors $|P_5 - P_4|$ trend toward 0 as displayed in Table 1. Therefore, Figure 6 already gives us the converged values of the instantaneous speed ratio.

When comparing Figure 3 through Figure 6, we can notice some profound differences. As already mentioned while discussing Table 1, we see that the curves of lower F_0 values, e.g. F_0 = 0.50 and F_0 = 0.70, converge slower than those of bigger F_0 values, e.g. F_0 = 1.00, 3.00 and F_0 = 5.00. Other than that, we also note that the curves for low values of easiness flow i.e. (F_0 = 0.50 and F_0 = 0.70) do not reach 1 in the middle of section length L, unlike

Instantaneous Speed Ratio and Error at Kilometer 31.6 from Shah Alam to Kuala Lumpur

Table 1
Number of iterations for instantaneous speed ratio when easiness to flow F_0 =0.50 and error $|P_n - P_{n-1}|$

X/L	P_1	P_2	P_3	P_4	P_5	$P_{6\rightarrow 10000}$	$\lvert P_2-P_1 \rvert$	$\lvert P_3-P_2 \rvert$	$\lvert P_4-P_3 \rvert$	$\lvert P_5-P_4 \rvert$
0.0	2.26E+00	2.54E+00	2.56E+00	2.56E+00	2.56E+00	2.56E+00	2.70E-01	2.22E-02	6.71E-04	7.45E-06
0.1	2.17E+00	2.37E+00	2.38E+00	2.38E+00	2.38E+00	2.38E+00	2.00E-01	9.90E-03	6.19E-05	2.04E-06
0.2	1.92E+00	1.95E+00	1.93E+00	1.93E+00	1.93E+00	1.93E+00	2.49E-02	1.34E-02	6.59E-04	6.34E-06
0.4	1.55E+00	1.39E+00	1.37E+00	1.37E+00	1.37E+00	1.37E+00	1.63E-01	2.18E-02	1.84E-04	5.51E-06
0.5	1.11E+00	8.49E-01	8.43E-01	8.44E-01	8.44E-01	8.44E-01	2.66E-01	6.08E-03	6.25E-04	3.32E-06
0.6	6.60E-01	4.30E-01	4.46E-01	4.47E-01	4.47E-01	4.47E-01	2.30E-01	1.64E-02	2.99E-04	7.32E-06
0.7	2.49E-01	1.75E-01	1.96E-01	1.95E-01	1.95E-01	1.95E-01	7.40E-02	2.07E-02	5.70E-04	6.87E-07
0.8	-6.34E-02	5.71E-02	5.91E-02	5.87E-02	5.87E-02	5.87E-02	1.20E-01	2.05E-03	4.04E-04	6.95E-06
0.9	-2.37E-01	-3.83E-03	-4.32E-03	-3.83E-03	-3.83E-03	-3.83E-03	2.34E-01	4.96E-04	4.96E-04	4.49E-06
1.0	-2.65E-01	5.25E-03	-1.70E-02	-1.63E-02	-1.63E-02	-1.63E-02	2.70E-01	2.22E-02	6.71E-04	7.45E-06

Table 2
Number of iterations for instantaneous speed ratio when easiness to flow F_0 =0.70 and error $|P_n - P_{n-1}|$

X/L	P_1	P_2	P_3	P_4	P_5	$P_{6\rightarrow 10000}$	$\lvert P_2-P_1 \rvert$	$\lvert P_3-P_2 \rvert$	$\lvert P_4-P_3 \rvert$	$\lvert P_5-P_4 \rvert$
0.0	2.05E+00	2.17E+00	2.17E+00	2.17E+00	2.17E+00	2.17E+00	1.21E-01	3.67E-03	2.73E-05	5.00E-08
0.1	1.97E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	2.06E+00	8.97E-02	1.64E-03	2.52E-06	1.40E-08
0.2	1.76E+00	1.77E+00	1.77E+00	1.77E+00	1.77E+00	1.77E+00	1.12E-02	2.21E-03	2.69E-05	4.30E-08
0.4	1.46E+00	1.38E+00	1.38E+00	1.38E+00	1.38E+00	1.38E+00	7.32E-02	3.61E-03	7.48E-06	3.70E-08
0.5	1.09E+00	9.75E-01	9.74E-01	9.74E-01	9.74E-01	9.74E-01	1.19E-01	1.00E-03	2.55E-05	2.23E-08
0.6	7.19E-01	6.16E-01	6.19E-01	6.19E-01	6.19E-01	6.19E-01	1.03E-01	2.71E-03	1.22E-05	4.94E-08
0.7	3.81E-01	3.48E-01	3.51E-01	3.51E-01	3.51E-01	3.51E-01	3.32E-02	3.42E-03	2.32E-05	4.60E-09
0.8	1.22E-01	1.77E-01	1.77E-01	1.77E-01	1.77E-01	1.77E-01	5.41E-02	3.39E-04	1.65E-05	4.68E-08
0.9	-2.27E-02	9.05E-02	8.74E-02	8.74E-02	8.74E-02	8.74E-02	1.13E-01	3.12E-03	2.02E-05	3.03E-08
1.0	-4.65E-02	7.49E-02	7.12E-02	7.13E-02	7.13E-02	7.13E-02	1.21E-01	3.67E-03	2.73E-05	5.02E-08

Table 3
Number of iterations for instantaneous speed ratio when easiness to flow F_0 =1.00 and error $|P_n - P_{n-1}|$

X/L	P_1	P_2	P_3	P_4	P_5	$P_{6\rightarrow 10000}$	$\lvert P_2-P_1\rvert$	$\lvert P_3-P_2\rvert$	$\lvert P_4-P_3\rvert$	$\lvert P_5-P_4\rvert$
0.0	1.79E+00	1.83E+00	1.83E+00	1.83E+00	1.83E+00	1.83E+00	3.66E-02	2.47E-04	2.25E-07	0.00E+00
0.1	1.73E+00	1.75E+00	1.75E+00	1.75E+00	1.75E+00	1.75E+00	2.70E-02	1.10E-04	2.10E-08	0.00E+00
0.2	1.57E+00	1.58E+00	1.58E+00	1.58E+00	1.58E+00	1.58E+00	3.37E-03	1.49E-04	2.21E-07	0.00E+00
0.4	1.34E+00	1.32E+00	1.32E+00	1.32E+00	1.32E+00	1.32E+00	2.20E-02	2.42E-04	6.20E-08	0.00E+00
0.5	1.07E+00	1.04E+00	1.03E+00	1.03E+00	1.03E+00	1.03E+00	3.59E-02	6.75E-05	2.10E-07	0.00E+00
0.6	7.89E-01	7.58E-01	7.58E-01	7.58E-01	7.58E-01	7.58E-01	3.11E-02	1.82E-04	1.00E-07	0.00E+00
0.7	5.35E-01	5.25E-01	5.25E-01	5.25E-01	5.25E-01	5.25E-01	1.00E-02	2.30E-04	1.91E-07	0.00E+00
0.8	3.40E-01	3.56E-01	3.56E-01	3.56E-01	3.56E-01	3.56E-01	1.63E-02	2.28E-05	1.36E-07	0.00E+00
0.9	2.29E-01	2.63E-01	2.63E-01	2.63E-01	2.63E-01	2.63E-01	3.41E-02	2.10E-04	1.66E-07	0.00E+00
1.0	2.09E-01	2.46E-01	2.46E-01	2.46E-01	2.46E-01	2.46E-01	3.66E-02	2.47E-04	2.25E-07	0.00E+00

Table 4
Number of iterations for instantaneous speed ratio when easiness to flow F_0 =3.00 and error $|P_n - P_{n-1}|$

X/L	P_1	P_2	P_3	P_4	P_5	$P_{6\rightarrow 10000}$	$\lvert P_2-P_1\rvert$	$\lvert P_3-P_2\rvert$	$\lvert P_4-P_3\rvert$	$\lvert P_5-P_4\rvert$
0.0	1.16E+00	1.16E+00	1.16E+00	1.16E+00	1.16E+00	1.16E+00	1.23E-05	0.00E+00	0.00E+00	0.00E+00
0.1	1.14E+00	1.14E+00	1.14E+00	1.14E+00	1.14E+00	1.14E+00	9.07E-06	0.00E+00	0.00E+00	0.00E+00
0.2	1.10E+00	1.10E+00	1.10E+00	1.10E+00	1.10E+00	1.10E+00	1.13E-06	0.00E+00	0.00E+00	0.00E+00
0.4	1.06E+00	1.06E+00	1.06E+00	1.06E+00	1.06E+00	1.06E+00	7.39E-06	0.00E+00	0.00E+00	0.00E+00
0.5	1.01E+00	1.01E+00	1.01E+00	1.01E+00	1.01E+00	1.01E+00	1.21E-05	0.00E+00	0.00E+00	0.00E+00
0.6	9.62E-01	9.62E-01	9.62E-01	9.62E-01	9.62E-01	9.62E-01	1.04E-05	0.00E+00	0.00E+00	0.00E+00
0.7	9.16E-01	9.16E-01	9.16E-01	9.16E-01	9.16E-01	9.16E-01	3.36E-06	0.00E+00	0.00E+00	0.00E+00
0.8	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01	8.77E-01	5.47E-06	0.00E+00	0.00E+00	0.00E+00
0.9	8.50E-01	8.50E-01	8.50E-01	8.50E-01	8.50E-01	8.50E-01	1.14E-05	0.00E+00	0.00E+00	0.00E+00
1.0	8.41E-01	8.41E-01	8.41E-01	8.41E-01	8.41E-01	8.41E-01	1.23E-05	0.00E+00	0.00E+00	0.00E+00

Table 5
Number of iterations for instantaneous speed ratio when easiness to flow F_0=5.00 and error $|P_n - P_{n-1}|$

X/L	P_1	P_2	P_3	P_4	P_5	$P_{6\rightarrow 10000}$	$\lvert P_2-P_1\rvert$	$\lvert P_3-P_2\rvert$	$\lvert P_4-P_3\rvert$	$\lvert P_5-P_4\rvert$
0.0	1.07E+00	1.07E+00	1.07E+00	1.07E+00	1.07E+00	1.07E+00	4.00E-09	0.00E+00	0.00E+00	0.00E+00
0.1	1.06E+00	1.06E+00	1.06E+00	1.06E+00	1.06E+00	1.06E+00	3.00E-09	0.00E+00	0.00E+00	0.00E+00
0.2	1.04E+00	1.04E+00	1.04E+00	1.04E+00	1.04E+00	1.04E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
0.4	1.02E+00	1.02E+00	1.02E+00	1.02E+00	1.02E+00	1.02E+00	3.00E-09	0.00E+00	0.00E+00	0.00E+00
0.5	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	1.00E+00	4.00E-09	0.00E+00	0.00E+00	0.00E+00
0.6	9.86E-01	9.86E-01	9.86E-01	9.86E-01	9.86E-01	9.86E-01	3.50E-09	0.00E+00	0.00E+00	0.00E+00
0.7	9.67E-01	9.67E-01	9.67E-01	9.67E-01	9.67E-01	9.67E-01	1.10E-09	0.00E+00	0.00E+00	0.00E+00
0.8	9.50E-01	9.50E-01	9.50E-01	9.50E-01	9.50E-01	9.50E-01	1.80E-09	0.00E+00	0.00E+00	0.00E+00
0.9	9.34E-01	9.34E-01	9.34E-01	9.34E-01	9.34E-01	9.34E-01	3.90E-09	0.00E+00	0.00E+00	0.00E+00
1.0	9.27E-01	9.27E-01	9.27E-01	9.27E-01	9.27E-01	9.27E-01	4.20E-09	0.00E+00	0.00E+00	0.00E+00

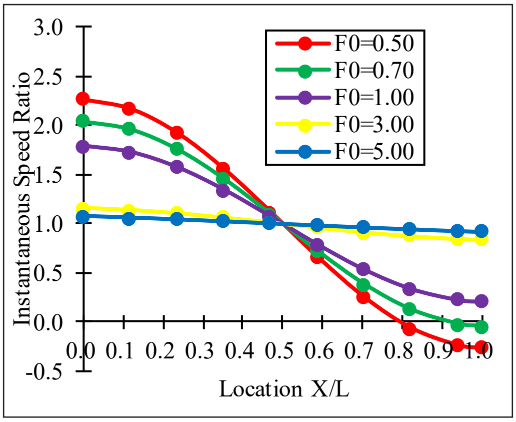

Figure 2. The instantaneous speed ratio versus location *X/L* which is plotted for the first iteration P_1 at every value of the easiness to flow parameter, F_0 = 0.50, 0.70, 1.00, 3.00 and 5.00.

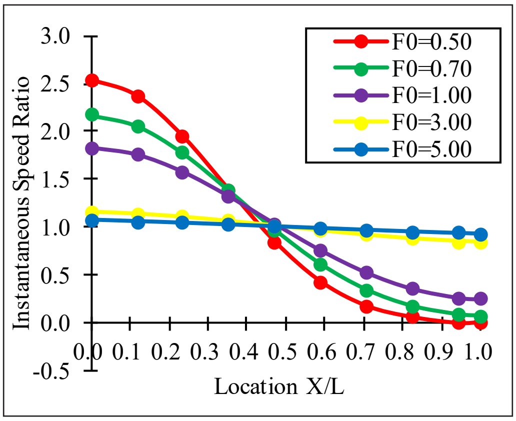

Figure 3. The instantaneous speed ratio versus location *X/L* which is plotted for the second iteration P_2 at every value of the easiness to flow parameter, F_0 = 0.50, 0.70, 1.00, 3.00 and 5.00.

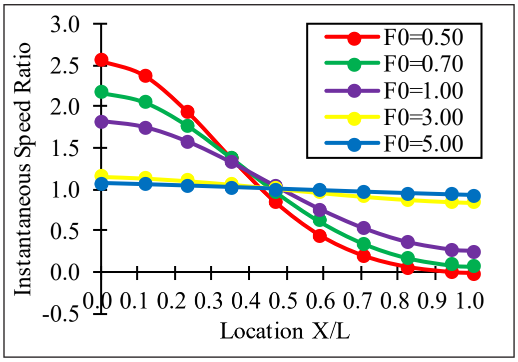

Figure 4. The instantaneous speed ratio versus location *X/L* which is plotted for the third iteration P_3 at every value of the easiness to flow parameter, F_0 = 0.50, 0.70, 1.00, 3.00 and 5.00.

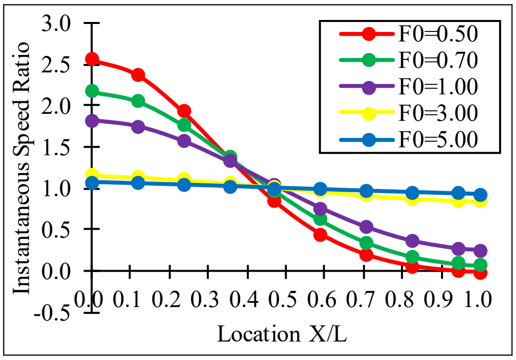

Figure 5. The instantaneous speed ratio versus location *X/L* which is plotted for the fourth iteration P_4 at every value of the easiness to flow parameter, F_0 = 0.50, 0.70, 1.00, 3.00 and 5.00.

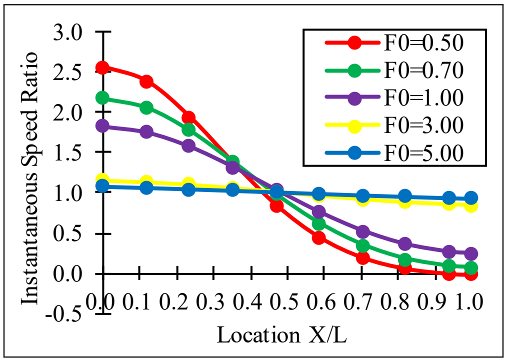

Figure 6. The instantaneous speed ratio versus versus location *X/L* which is plotted for the fifth iteration P_5 at every value of the easiness to flow parameter, F_0 = 0.50, 0.70, 1.00, 3.00 and 5.00.

the curves of bigger values of F_0. Instead, the curves reach 1 for instantaneous speed ratios earlier when the values of F_0 are small, e.g. $F_0 = 0.50$ and $F_0 = 0.70$. This is due to the necessity for drivers to slow down their vehicles for at least 1/5 of $L/2$ to avoid collision.

In our numerical experiments, *1/5* of *L/2* takes the value $X/L = 0.4$ from the starting point. From Figure 6, we observe that the realistic data taken on the ramp at kilometer 31.6 from Shah Alam to Kuala Lumpur fulfills this minimum necessity to avoid collision. We can see that at $X = 0.4$, the instantaneous speed ratio is already almost 1 for all the values of easiness to flow F_0.

We attempted to calculate the instantaneous speed ratio to infinite iterations such that $n = \infty$ but this failed due to the limitations of our computer memory. The greatest number of iterations n that we could achieve in generating the results for P_n was $n = 10000$. However, the results of the instantaneous speed ratio when $n = 10000$ are not much different graphically (Figure 6) compared to when $n = 5$.

The x-axis of Figure 7 is the dependent variable of the ratio X/L which takes the values $0 \leq X/L \leq 1$. In our numerical experiments, $L = 170$ meters. If we compare the graph of our converged numerical results in Figure 6 with the theoretical results in Figure 7, we notice several similarities. We observe that for higher F_0 values such as $F_0 \rightarrow 5.00$, the instantaneous speed ratio is more stable at 1. This is due to the fact that the vehicles slow down due to heavy traffic.

For the lower traffic flow that is when we have lower values of easiness to flow, the theory suggests that the vehicles to slow down to about *1/5* from the merging point at *L/2*. Therefore, the vehicles need to slow down and the instantaneous speed ratio is expected to reach toward 1 at $X/L = 0.4$. If we look at the theoretical graph in Figure 7, the instantaneous speed ratio is ≤ 1.4 when $X/L = 0.4$. From our realistic graph in Figure 6, we observe that the instantaneous speed ratio is ≤ 1.34 when $X/L = 0.4$ which is acceptable for avoiding collision.

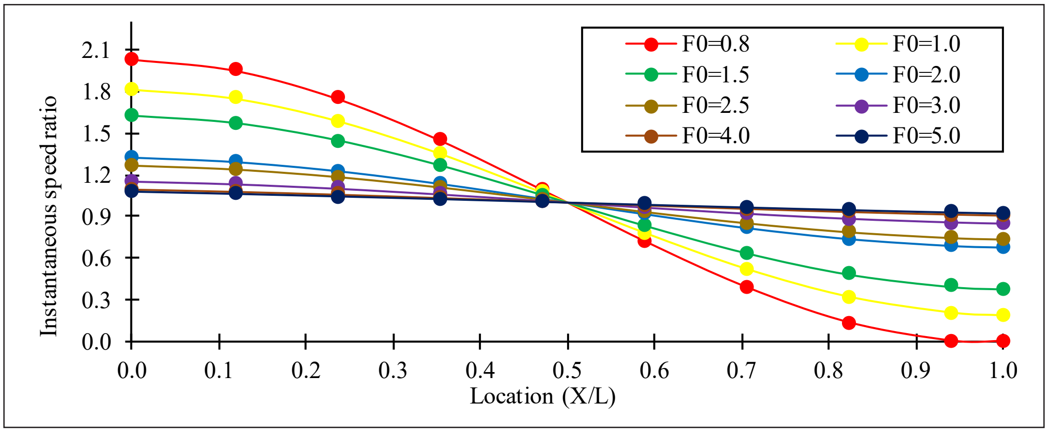

Figure 7. Theoretical speed ratio–distance graph by Reddy (1966)

CONCLUSION

In this paper, the model that we took into consideration was the continuous flow model which was established on the idea that traffic flow was analogous to the flow of a one-dimensional fluid or gas. The model is analogous to the model by Reddy (1966). Unlike Reddy (1966), whose model represented left hand drive traffic, the model that we solved here was for right hand drive traffic. We solved the model, an IBVP, and the solution was formed in terms of a Fourier series which was also denoted as the instantaneous speed ratio.

We evaluated the continuous flow model by measuring the instantaneous speed ratio on the ramp at kilometer 31.6 from Shah Alam to Kuala Lumpur. On the whole, the instantaneous speed ratio at lower values of the easiness to flow parameter, $F_0 = 0.50$ and $F_0 = 0.70$, shows slower convergence compared to when we have higher values of F_0, i.e. $F_0 = 1.00$, 3.00 and $F_0 = 5.00$.

In a nutshell, the one-dimensional viscous flow of compressible fluid theory provides a good theoretical model to explain the variations of instantaneous speed ratio on the merging area of a highway due to the traffic conditions. Other than that, this study can help us to find improvements to the design of the entrance ramp.

One of the suggestions is to provide a sufficient length for acceleration at the entrance ramp. It is useful to evaluate the reasonableness of the parameters and analyze their numerical values in order to enhance the efficiency of the traffic flowing at the merging area on highways.

In our studies, we deduce that the instantaneous speed ratio on the ramp at kilometer 31.6 from Shah Alam to Kuala Lumpur is acceptable since as $X/L \rightarrow 0.4$, the instantaneous speed ratio ≤ 1.34 which is less than 1.4 as suggested by Reddy (1966) as in the theoretical graph in Figure 7. Therefore, minimal collisions are expected due to the well-designed ramp at kilometer 31.6 from Shah Alam to Kuala Lumpur.

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